

An Investigation of Atmospheric Loss in a Thicker Martian Atmosphere

Team 322

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Martian atmospheric has long been at the center of interplanetary studies, as it contains important information regarding the formation of Solar system at its early stages. The atmosphere of Mars, largely consisted of CO_2 , poses puzzles about the absence of other lighter elements. In this paper, we used an heuristic approach to investigate the loss of atmospheric constituents by constructing a physical model of Martian atmosphere based on principle laws of physics. With reasonable assumptions, an expression of atmospheric pressure is obtained, from which we determine the time needed for Martian surface pressure to decrease from 0.2 to 0.1 to be 6.2 billion years. An exponential relationship between pressure, time, and altitude was also well-established.

[Problem A - UPC 2021]

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I. INTRODUCTION

A. Approach

As Benjamin Franklin wrote, “A small leak can sink a great ship.” Present-day observations of our universe are the result of billion years of complex evolution. Mars has long captured the attention of scientific community, as it poses multiple questions about its properties. Martian geomorphology appears to indicate that there was once the existence of abundant water on Mars’ surface, which is now a vast region of basalt.

In this paper, we consider a scenario in which the polar caps and subsurface ice had been vaporized. As a result, an enormous amount of carbon dioxide was released to the Martian atmosphere. Consequently the atmosphere pressure on Mars’s surface will increase to 0.2 bar. Because of atmosphere escape, the loss of planetary atmospheric gases to outer space, the atmosphere of Mars would decrease over time. A heuristic model of Martian atmosphere is constructed. By expression of atmospheric pressure as a function of time, the escape of gases to outer space is computed. Therefore, the amount of time for Martian atmospheric pressure decrease to 0.1 bar is calculated.

B. General Assumptions

1. Atmosphere of Mars

Martian atmosphere is extremely thinner than at of the Earth. The average surface pressure of Mars not only varies due to elevation but also depends on the evaluation time since the amount of CO_2 gas in the atmosphere changes with the seasons¹. In our proposed model, the average surface pressure p_0 is initially set to 0.2 bar. Changes of the seasons is neglected as an attempt to make the model less complicated. The decrease in the average pressure on the surface over time is then the re-

sults of the escape process of atmospheric gases to outer space.

TABLE I. Atmospheric composition of Mars. The remaining 0.17% constituents are negligible since their disappearance is irrelevant.

Composition	By volume
Carbon dioxide CO_2	95.32%
Nitrogen N_2	2.7%
Argon Ar	1.6%
Oxygen O_2	0.13%
Carbon Monoxide CO	0.08%

The Martian atmosphere is considered to contain more CO_2 as a result of vaporization of the polar caps and subsurface ice. To simplify the model, we assumed that the Martian atmosphere is exclusively composed of CO_2 . This assumption is reasonable, since current data has already showed that CO_2 makes up 95% Martian atmosphere. It is also assumed that the increase of CO_2 in the atmosphere directly enhance the greenhouse effect; hence, initially Mars is heated to a temperature T_0 of 286 Kelvin from 215 Kelvin—the initial average temperature of Martian atmosphere. We will discuss further this temperature assumption in the next section.

2. Escape Mechanisms

The escape of atmospheric gases in all planet are based on the same mechanisms, which includes thermal escape, non-thermal escape and impact erosion. To simplify the model, we assume no significant asteroid impact would occur, thus rule out the contribution of such events.

a. Thermal escape. When exospheric molecules (favourably light ones) reach a sufficiently high velocity, they will have enough energy to escape to interplanetary space. There are two types of thermal escape, one is Jean escape, the other is hydrodynamic.

Due to Maxwellian velocity distribution, a portion of atoms/molecules, travelling in exospheric region (where their mean free path is comparable in length to the pressure scale height), will have $v > v_{esp}$, and be able to escape from Mars under Jean escape mechanism. The Jean's escape velocity v_s can be found by integrating over the Maxwellian velocity distribution, and multiplied by the number density n_c yields the escape flux

$$\phi_{esc} = \frac{1}{2\pi} n_c v_s (1 + \lambda_c) e^{-\lambda_c} \quad (1)$$

where λ_c is the escape parameter, defined as the ratio of gravitational potential energy to thermal energy at the critical level. We note that the Martian atmosphere primarily consists of heavy elements, thus loss of its constituents due to Jean escape is neglected. However, early stages of Martian atmospheric model would otherwise be incomplete if we abandon the stream of light molecules/atoms leaving by Jean's mechanism.

Hydrodynamic escape, on the other hand, based on the temperature of the exosphere. The higher the exospheric temperature, the more atoms/molecules escaping hydrodynamically. However, upper atmosphere of Mars is relatively cold since it is dominated by CO₂, a good IR radiator—around 200 K, given the enhanced greenhouse effect of ground level; therefore, atmospheric loss resulted from hydrodynamic escape is also ruled out.

b. Non-thermal escape. There are four types of non-thermal escape: photochemical, charge exchange, sputtering, and polar wind escape. These processes take place due to photochemistry or ionized atoms interactions.

Given that NASA's MAVEN (Vaisberg, 2018) data disproved the existence of the magnetosphere of Mars (its magnetic field is intrinsically weak), only photochemical and sputtering escape, which are even strengthened by the absence of such region, are of our interests.

II. THEORETICAL ANALYSIS

Since we wish to know how long will the Martian atmospheric pressure p reduces to 0.1 bar, our goal is to obtain an expression of $p(t)$ as a function of time. This relationship is based on the rate at which molecules are leaving the planet dN/dt . Furthermore, the air density is Maxwellian, thus $p(t, z)$ must also vary with height.

A. Barometric Pressure Formula

Using the laws of ideal gas, we obtain the atmospheric pressure $p(t, h)$ dependent on the temperature $T(t)$ in the atmosphere and the height h

$$p(t, h) = p_0(t) \exp \left\{ -\frac{ghM}{RT(t)} \right\} \quad (2)$$

where p_0 is the initial pressure of 0.2 bar, g is the gravitational acceleration constant on Mars, $R = 8.314 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ is the universal gas constant, and h is the height of the considered point.

B. Particles escaping rate

1. The initial total number of molecules in Martian atmosphere

We calculate the volume of Martian atmosphere by assuming that the atmosphere is a spherical section with a width of the scale height. Then the Martian atmosphere volume is

$$V_0 = \frac{4}{3}\pi \{(R+h)^3 - R^3\} \quad (3)$$

where $R = 3389.5 \text{ km}$, H is defined as the pressure scale height of Mars 11.1 km. The volume of Martian atmosphere is $V = 1.6 \times 10^9 \text{ km}^3$. According to the average mass density of air in Mars 0.02 kg/m^3 , the mass of Martian atmosphere is given by

$$m_0 = V_0 \cdot D = 3.2 \times 10^{16} \text{ kg} \quad (4)$$

From observations, a thick amount of water ice was covered under a surface layer of solid CO₂. The North pole of Mars has a diameter of 350 km and thickness of 8 m. Meanwhile, the South pole has a diameter of 1100 km and height of 1 m. The volume of solid carbon dioxide is $V = 1.72 \times 10^{12} \text{ m}^3$. Because the density of solid carbon dioxide is $\rho = 1562 \text{ kg/m}^3$, the mass of the carbon dioxide from polar caps emitted to the Martian atmosphere is given by

$$m = V\rho = 2.69 \times 10^{15} \text{ kg} \quad (5)$$

Hence, the total mass of carbon dioxide (initial value plus the newly emitted) in the atmosphere of Mars is

$$m_{tot} = m_0 + m = 3.469 \times 10^{16} \text{ kg} \quad (6)$$

Whereas, one mole of carbon dioxide weights $\rho_m = 44 \text{ grams}$, since we have assumed that Martian atmosphere contains only carbon dioxide, the total number of molecules in Martian atmosphere is

$$N_{tot} = \frac{m_0}{\rho_m} \cdot N_A = 4.75 \times 10^{41} \text{ particle} \quad (7)$$

The number of molecules CO₂ remaining after t second(s) is

$$N(t) = N_{tot} - \frac{dN}{dt} dt = f(T_0) \quad (8)$$

where dN/dt is the rate at which atmospheric loss take place. From literature (MAVEN, 2015) and above assumptions, we have decided that the main driver of atmospheric loss is sputtering escape.

C. The relationship between the number of molecules $N(t)$ and temperature $T_0(t)$

The concept of optical depth τ is now introduced. It establishes the connection between the number of molecules N in a region and its temperature T . The ratio is given by

$$\frac{\tau_N(\text{CO}_2)}{\tau_{N_0}(\text{CO}_2)} = \frac{N}{N_0} \quad (9)$$

Since $\tau(\nu) = f(T_0)$, the relationship between the number of molecules $N(t)$ and $T(h_0, t)$ is then well-established. Now that we have a function of temperature with respect to time, $p(z, t)$ is now obtained. Furthermore, together $\tau(\nu) = f(T_0)$ and (9) allows us to calculate the aforementioned temperature 286 K above (Miyazaki).

D. Relationship between Average Pressure and Number of Molecules

Average surface pressure on the ground of Mars is

$$p_0 = \frac{F}{S} = \frac{1}{4\pi R^2} \int_0^{R_a-R} dh N(h) g(h) M_{\text{CO}_2} \quad (10)$$

where h is the height from the ground of Mars, R is the radius of Mars, R_a is the radius of the boundary of Martian atmosphere, and $N(h)$ is the number of molecule per square meter at the height h (molecules/km²).

Moreover, the number of molecule per square meter at the height h is expressed as

$$N(h) = n(h) 4\pi(R+h)^2 \quad (11)$$

$$= 4\pi(R+h)^2 n_0 \exp\left\{-\frac{h}{H}\right\} \quad (12)$$

where n_0 is the molecule density (molecule/km³). Let us assume an vertically uniform gravitational acceleration g . The pressure p_0 is then determined by

$$p_0 = \frac{n_0}{4\pi R^2} dh \int_0^{R_a-R} 4\pi R^2 g M_{\text{CO}_2} \exp\left\{-\frac{h}{H}\right\} dh \quad (13)$$

It is evident that the integral in (13) is a constant, independent of h . Therefore, P_0 is directly proportional to n_0 . In other words,

$$\frac{p_0}{n_0} = k \in \mathbb{R}^* \quad (14)$$

The total number of molecules at t is

$$N(t) = 4\pi R^2 \int_0^{R_a-R} dh n(t, h) \quad (15)$$

$$= 4\pi R^2 \int_0^{R_a-R} dh n_0(t) \exp\left\{-\frac{h}{H}\right\} \quad (16)$$

$$= 4\pi R^2 n_0(t) \int_0^{R_a-R} dh \exp\left\{-\frac{h}{H}\right\} \quad (17)$$

From the above equations, the total number of molecules at a given time t is in direct proportion to the molecule density $n_0(t)$. Therefore, the average surface pressure is in ratio to the total number of molecules:

$$p_0(t) \propto N(t) \quad (18)$$

III. MODEL ANALYSIS & RESULTS

A. Time needed for p_0 to reduce to 0.1 bar

Let us now compute how long would p_0 need to decrease from 0.2 bar to 0.1 bar. The proportionality in Eqn. (18) implies that

$$\frac{N(t)}{N(t=0)} = \frac{P_0(t)}{P_0(t=0)} = \frac{0.1}{0.2} = \frac{1}{2}$$

$$N(t) = N(0) - \frac{dN}{dt}t = \frac{N_0}{2} \leftrightarrow \frac{dN}{dt}t = \frac{N_0}{2}$$

where $dN/dt \approx 2.4 \times 10^{24}$ molecules/s and $N_0 \approx 4.75 \times 10^{41}$ molecules. Hence,

$$t \approx 6.2 \text{ billion years}$$

B. Barometric Pressure Formula Modified

The relationship implied in Eqn. (14) can be utilised further. We now represent p_0 in its relationship with the number of molecules at a given time t . The modified $p(h, t)$ from (2) is now comprehensively describe how pressure changes with respect to height and time,

$$p(h, t) = p_0 \frac{N(t)}{N_0} \exp\left\{-\frac{ghM}{RT(t)}\right\} \quad (19)$$

where p_0 , N_0 is the initial surface pressure and number of CO₂ molecules of our model, respectively; $N(t)$ is the number of CO₂ at time t . Figure 1 illustrates the variation of pressure over time and height. Initial values are $T_0 = 286$ K, $p_0 = 0.2$ bar, $g = 3.721$ m/s². Our simulation concentrates primarily in range of $0 \leq h \leq 7$ km. From 10 km and beyond, the temperature is mainly manipulated by solar activities.

IV. CONCLUSION

A. Discussion of the results

1. Time Needed

Our model estimated a period of 6.2 billion years for the decrease of Martian surface pressure from 0.2 to 0.1 bar. This values is highly oscillatory since we assume an seemingly straightforward set of values for dN/dt and N_0 . We also notice that, given the fact that Martian surface pressure was initially 0.2 bar, the number of CO₂ calculated directly from this assumption yields a significantly larger result compared to our chosen value (4.75×10^{41} particle) by a factor of 10, which corresponds to a total time needed of 62 billion years.

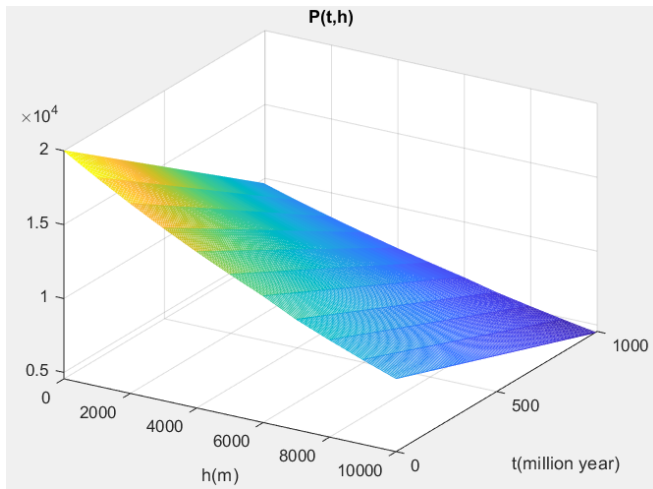


FIG. 1. Variation of pressure $p(t, h)$ with height h (in meters) and t (in million years)

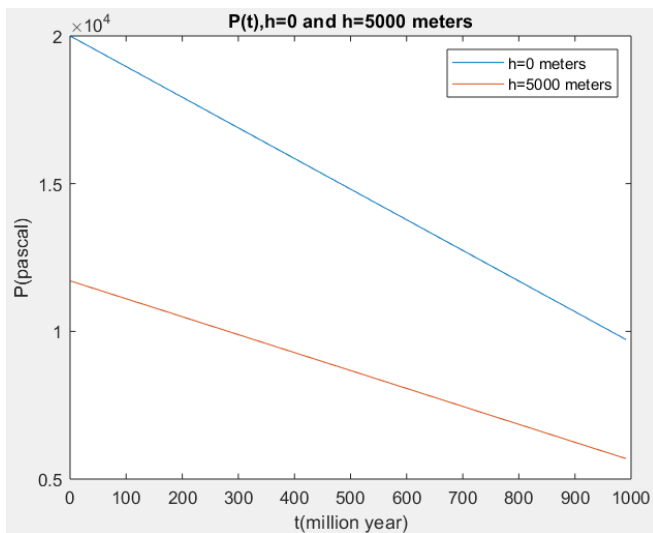


FIG. 2. How p changes with respect to time t .

2. Variation of Barometric Pressure

When considering the dependence of Martian average pressure on time, we got Figure (2). As shown in Figure (2), the average pressure at one position of altitude is linear to the amount of time. Moreover, the data at $h = 0$ experienced a more significantly decrease than that with $h = 5000$ km.

As shown in Figure (2), it is clearly that the average pressure varies with the height. Initially, when the average pressure of Mars is 0.2 bar at a height of 0 km, the average pressure decline by approximately a half to 0.1 bar at a height of 10 km. This change is considered as an exponential function of height h . After 6.2 billion years, the average pressure of Mars decreased to 0.2 bar, this figure still conform to an exponent function. The average pressure on Mars³ was expressed as a function of height

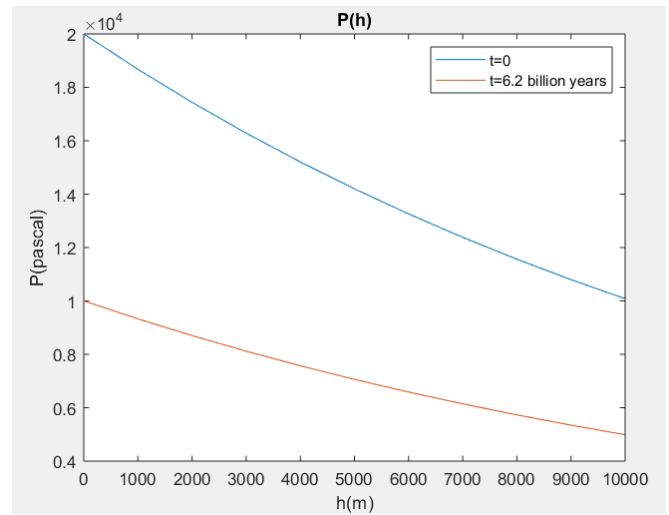


FIG. 3. How p changes with respect to altitude h .

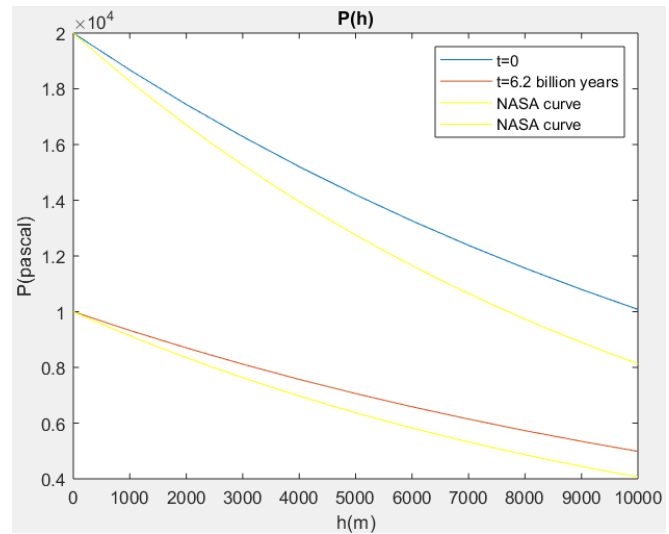


FIG. 4. Predicted p in comparison with NASA data²

from the ground

$$p = 0.699 \cdot \exp \{-0.00009 \cdot h\} \quad (20)$$

where $p_0 = 0.699 \times 10^5$ Pa is the average pressure on the ground of the practical model of Mars. The pressure decreased when getting higher. Therefore, our proposed model is relative reasonable when comparing with the practical Mars.

To verify our study, we plotted the graph of the average pressure followed Equation (19) and replaced $p_0 = 0.2$ bar $= 0.2 \times 10^5$ for $t = 0$ and $p_0 = 0.1$ bar $= 0.1 \times 10^5$ for $t = 6$ billion years and the function of our model in the same graph, as shown in Figure (4). According to Figure (4), although there were differences between two models, we can conclude that our design is rational with the experimental results. This is a reasonable difference, given the coarseness and uncertainties of our model.

B. Strength and Weakness

Our study succeeded in finding the relationship between the Martian average pressure and time, which directly lead to the answer for the question how long does it take until to the pressure is only 0.1 bar, equal to a half of initial value 0.2 bar.

Moreover, comparison with the consequence of NASA indicated that our proposed theory is reasonable, since we were able to replicate the profiles of experimental results obtain on Mars.

Aside from these strengths, there are several setbacks in our proposed model. We have ruled out some possible

mechanisms by which atmospheric loss can take place. Moreover, the choice of the rate at which sputtering escape happens is highly based on past literature, which is not comprehensive and complete. Last, our predicted numbers of CO₂ in the atmosphere using two different approaches show great contrast. In addition, we had to assume multiple parameters such as gravitational acceleration g and scale height H , which change with time, as constants.

APPENDIX A: CODE

```
clc; clear all; close all;
dt=10; t=1:dt:1000;T_b=215;r=5.186e-4;P0=20000;
N=length(t); dh=1e3;h=0:dh:10e3;
T=zeros(length(t),length(h));
P=zeros(length(t),length(h));
for i=1:length(t)
    N(i)=1-t(i)*r;
    a = 1.5*N(i)*54.45;
    T(i,1)=sqrt(sqrt((a+1)*(215^4)/(0.3*a+2)));
    for j=1:length(h)-1
        if(j<7)
            T(i,j+1)=T(i,1)-0.000998*(h(j+1));
        end
        if(j>=7)
            T(i,j+1)=T(i,1)-0.00222*(h(j+1));
        end
    end
    for j=1:length(h)
        P(i,j)=P0*(1-r*t(i))*exp((-3.7*0.044*h(j))/(T(i,j)*8.31));
        P1(j)=P0*exp((-3.7*0.044*h(j))/(286*8.31));
        P2(j)=P0/2*exp((-3.7*0.044*h(j))/(281.5*8.31));
        P4(i)=P0*(1-r*t(i));
        P3(i)=P0*(1-r*t(i))*exp((-3.7*0.044*5000)/(183*8.31));
    end
end
figure(1);mesh(h,t,P);view([30 35]);
hold on; title('P(t,h)')
ylabel('t(million year)')
xlabel('h(m)')
hold off;
figure(2);mesh(h,t,T);view([30 35]);
hold on; title('T(t,h)')
ylabel('t(million year)')
xlabel('h(m)')
hold off;
figure(3),plot(h,P1),title('P(h)'),xlabel('h(m)'),ylabel('P(pascal)'),
hold on
plot(h,P2)
fplot(@(x) 20000*exp(-0.00009*x),[0 10000], 'y')
xlabel('h(m)'),ylabel('P(pascal)')
```

```

fplot(@(x) 10000*exp(-0.00009*x),[0 10000], 'y')
xlabel('h(m)'),ylabel('P(pascal)')
legend({'t=0', 't=6.2 billion years', 'NASA curve', 'NASA curve'})
hold off

figure(4),plot(t,P4),title('P(t),h=0 and h=5000 meters'),
xlabel('t(million year)'),
ylabel('P(pascal)'),
hold on
plot(t,P3)
legend({'h=0 meters', 'h=5000 meters'})
hold off

```

References

¹<https://marsed.asu.edu/mep/atmosphere>

²<https://www.grc.nasa.gov/www/k-12/airplane/atmosmrm.html>

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